

CURRICULUM VITAE

CARLO SANNA

Contact information

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Positions

01/03/2022 | **Researcher (RTDb)**
– on going | Politecnico di Torino
01/02/2020 | **Postdoctoral Researcher**
– 31/01/2022 | Politecnico di Torino
01/02/2019 | **Postdoctoral Researcher**
– 31/01/2020 | Unità INdAM dell’Università di Genova

Abilitazione

Abilitazione Scientifica Nazionale per il ruolo di Professore di Seconda Fascia, Settore Concorsuale 01/A2 (Geometria e Algebra), valida dal 11/11/2020 al 11/11/2031.

Education

15/10/2018 | **PhD in Pure and Applied Mathematics**, cum laude
Università degli Studi di Torino, in association with Politecnico di Torino
Thesis title: *“Arithmetic properties of linear recurrences and other topics in Number Theory”*
Supervisor: Prof. Danilo Bazzanella
Referees: Prof. Giuseppe Molteni, Prof. Alberto Perelli
16/07/2015 | **Master of Science, Mathematics**, 110/110 cum laude and “menzione di merito”
Università degli Studi di Torino
15/10/2013 | **Bachelor of Science, Mathematics**, 110/110 cum laude
Università degli Studi di Torino

Awards, prizes, and scholarships

2020 | **Award con.Sienze** (€ 1.000)
sponsored by “Conferenza Nazionale dei Presidenti e dei Direttori delle Strutture Universitarie di Scienze e Tecnologie”, and given to the six best PhD theses in the scientific areas CUN 01-05 (08/2018–07/2019)
2016 | **Award Medaglia d’Argento and prize Luciana Picco Botta** (€ 2.000)
sponsored by Università degli Studi di Torino,
for the best master thesis in Mathematics of the academic year 2014/15

2015 | **PhD scholarship “Contemporary Problems in Mathematics”**
Ranked first over 35 candidates

Research groups and projects

2023 – 2026 | **Quantum-oriented Update to Browsers and Infrastructure for the Post-Quantum Transition (QUBIP) – HORIZON-CL3-2022-CS-01-03**
Responsabile Scientifico di Struttura

2023 – 2025 | **SEcurity and RIghts in the CyberSpace (SERICS) – PNRR, Mission 4**
Responsabile Scientifico Spoke 5

2018
– ongoing | **Member of CryptTO – Group of Cryptography and Number Theory**
Department of Mathematical Sciences “G. L. Lagrange”, Politecnico di Torino

2018
– ongoing | **Member of “Gruppo Nazionale per le Strutture Algebriche, Geometriche e le loro Applicazioni”**
Istituto Nazionale di Alta Matematica

2017 – 2018 | **Member of the research project “Algebra e Dintorni”**
Department of Mathematics “G. Peano”, Università degli Studi di Torino

2017 – 2018 | **Member of the research project “Approssimazione multivariata e algoritmi efficienti con applicazioni a problemi algebrici, differenziali e integrali”**
Department of Mathematics “G. Peano”, Università degli Studi di Torino

Conferences

22/09/2021 | **Co-organizer of “6th Number Theory Meeting – Torino”**
– 23/09/2021 | Department of Mathematical Sciences “G. L. Lagrange”, Politecnico di Torino

26/10/2021 | **Co-organizer of “5th Number Theory Meeting – Torino”**
– 27/10/2021 | Department of Mathematical Sciences “G. L. Lagrange”, Politecnico di Torino

27/05/2021 | **Co-organizer of “CryptTO Conference 2021”**
– 28/05/2021 | Department of Mathematical Sciences “G. L. Lagrange”, Politecnico di Torino

24/10/2019 | **Co-organizer of “4th Number Theory Meeting – Torino”**
– 25/10/2019 | Department of Mathematics “G. Peano”, Università di Torino

15/10/2018 | **Co-organizer of “3rd Number Theory Meeting – Torino”**
– 16/10/2018 | Department of Mathematics “G. Peano”, Università di Torino

26/10/2017 | **Co-organizer of “2nd Number Theory Meeting – Torino”**
– 27/10/2017 | Department of Mathematical Sciences “G. L. Lagrange”, Politecnico di Torino

Teaching experiences

2022/23 | **“Linear Algebra and Geometry”** (60 hours)
Politecnico di Torino

2021/22 | **“Linear Algebra and Geometry”** (60 hours)
Politecnico di Torino

2020/21 | **Assistant of “Linear Algebra and Geometry”** (40 hours)
Politecnico di Torino

2019/20	PhD course “Blockchain and Cryptoeconomy” (first part, 10 hours) Politecnico di Torino
2018/19	Assistant of “Mathematical Methods for Engineers” (40 hours) Politecnico di Torino
2018/19	Assistant of “Mathematics I” (40 hours) Corso di Laurea Triennale in Chimica e Tecnologie Chimiche, Università di Torino
2017/18	Assistant of “Mathematical Analysis I” (60 hours) Politecnico di Torino
2016/17	Assistant of “Mathematical Analysis I” (60 hours) Politecnico di Torino

Thesis supervisor

15/03/22	Sistemi Post-Quantum: Crittografia Multivariata Laurea Magistrale in Ingegneria Matematica, Politecnico di Torino Candidate: Adriano Koleci, Supervisors: Danilo Bazzanella, Carlo Sanna
23/11/21	Numeri Pseudocasuali: Proprietà e Applicazioni in Crittografia Laurea in Matematica per l’Ingegneria, Politecnico di Torino Candidate: Davide Arnaudo, Supervisors: Danilo Bazzanella, Giuseppe D’Alconzo, Carlo Sanna
11/09/20	Cifrari Monoalfabetici e Analisi delle Frequenze Laurea in Matematica per l’Ingegneria, Politecnico di Torino Candidate: Matteo Galla, Supervisors: Danilo Bazzanella, Carlo Sanna

Talks and presentations

26/05/23	Schemi di Firma Digitale Post-Quantum Basati su MinRank CrypTO Conference, Torino
04/05/23	Zeckendorf expansions of multiplicative inverses The 7th mini symposium of the Roman Number Theory Association, Rome
05/01/23 (invited)	Some results on the greatest common divisors of terms of linear recurrences Faculty of Mathematics and Computer Science of the Jagiellonian University, online
06/12/22 (invited)	Some results on the greatest common divisors of terms of linear recurrences Dipartimento di Matematica, Università di Genova
26/05/22	Membership in random ratio sets <i>Combinatorial and Additive Number Theory 2022</i> , New York Number Theory Seminar, online
11/12/21	On the number of residues of linear recurrences <i>Combinatorial Number Theory and Connected Topics (CONTACT-I)</i> , online
26/05/21	Additive bases and Niven numbers <i>Combinatorial and Additive Number Theory 2021</i> , New York Number Theory Seminar, online
04/03/21 (invited)	Least common multiples of integer sequences: old and new results <i>Number Theory Online</i> , Università di Pisa, Roma Sapienza, Parma, online
18/02/21 (invited)	Cryptanalysis of Multivariate Public Key Cryptosystems <i>Technology Innovation Institute</i> , United Arab Emirates, online

- 05/02/21 **Introduction to Multivariate Cryptography**
(invited) *Seminario della serie De Cifris Augustae Taurinorum*, online
- 23/09/20 **Additive and Xor differential probabilities of Chacha quarter round**
(invited) *Technology Innovation Institute*, United Arab Emirates, online
- 22/07/20 **On the least common multiple of random terms of binary recurrence sequences**
19th International Conference on Fibonacci Numbers and Their Applications, online
- 21/05/19 **On the number of distinct exponents in the prime factorization of an integer**
(invited) *Seminari di Algebra e Geometria Algebrica*, Università di Torino
- 12/04/19 **Approximations by signed harmonic sums and the Thue–Morse sequence**
5th mini symposium of the Roman Number Theory Association, Università Roma Tre
- 18/04/18 **Waring’s theorem for binary powers**
4th mini symposium of the Roman Number Theory Association, Università Roma Tre
- 12/04/18 **Distribution of integral values for the ratio of two linear recurrences**
(invited) *Giornata dei dottorandi di Teoria dei Numeri*, Università degli Studi di Parma
- 19/12/17 **A coprimality condition on consecutive values of polynomials**
(invited) *Giornate INdAM di Teoria dei Numeri*, Università degli Studi di Genova
- 21/09/17 **On the greatest common divisor of n and the n th Fibonacci Number**
20th International Workshop for Young Mathematicians, Jagiellonian University
- 06/07/17 **On the G.C.D. of n and the n th term of a linear recurrence**
XXXth Journées Arithmétiques, University of Caen
- 05/06/17 **On the sum of digits of the factorial**
Numeration 2017, Università degli Studi Roma Tre
- 04/11/16 **A factor of integer polynomials with minimal integrals**
1st Number Theory Meeting - Torino, Università degli Studi di Torino

Preprints

On the distribution of the entries of a fixed-rank random matrix over a finite field

<https://arxiv.org/abs/2307.14172>

On the inverse of a Fibonacci number modulo a Fibonacci number being a Fibonacci number

<http://arxiv.org/abs/2306.08382>

Smaller public keys for MinRank-based schemes

(with Antonio J. Di Scala)

<https://arxiv.org/abs/2302.12447>

Toward a Post-Quantum Zero-Knowledge Verifiable Credential System for Self-Sovereign Identity

(with Simone Dutto, Davide Margaria, and Andrea Vesco)

<https://eprint.iacr.org/2022/1297>

On the index of appearance of a Lucas sequence

<https://arxiv.org/abs/2212.06127>

Accepted peer-reviewed papers

RLWE and PLWE over cyclotomic fields are not equivalent

(with Antonio J. Di Scala and Edoardo Signorini)

Applicable Algebra in Engineering, Communication and Computing

<https://doi.org/10.1007/s00200-022-00552-9>

Finding differential trails on ChaCha by means of state functions

(with Emanuele Bellini, Juan Grados, and Rusydi H. Makarim)

International Journal of Applied Cryptography

Published peer-reviewed papers

- 2023 **On the greatest common divisor of n and the n th Fibonacci number, II**
(with Abhishek Jha)
Canadian Mathematical Bulletin **66**, 617–625
<https://doi.org/10.4153/S0008439522000595>
- 2023 **A note on the distribution of weights of fixed-rank matrices over the binary field**
Finite Fields and Their Applications **87**, 102157
<https://doi.org/10.1016/j.ffa.2022.102157>
- 2023 **Zeckendorf representation of multiplicative inverses modulo a Fibonacci number**
(with Gessica Alecci and Nadir Murru)
Monatshefte für Mathematik **201**, 1–9
<https://doi.org/10.1007/s00605-022-01724-y>
- 2023 **Pairwise modular multiplicative inverses and Fibonacci numbers**
INTEGERS **23**, article A3
<http://math.colgate.edu/~integers/vol23.html>
- 2022 **MR-DSS – Smaller MinRank-based (Ring-)Signatures**
(with Emanuele Bellini, Andre Esser, and Javier Verbel)
PQCrypto 2022: Post-Quantum Cryptography
Lecture Notes in Computer Science, vol. 13512, pp. 144–169
https://doi.org/10.1007/978-3-031-17234-2_8
- 2022 **An Estimator for the Hardness of the MQ Problem**
(with Emanuele Bellini, Rusydi H. Makarim, and Javier Verbel)
Progress in Cryptology—AFRICACRYPT 2022
Lecture Notes in Computer Science, vol. 13503, pp. 323–347
https://doi.org/10.1007/978-3-031-17433-9_14
- 2022 **A survey on coefficients of cyclotomic polynomials**
Expositiones Mathematicae **40**, 469–494
<https://doi.org/10.1016/j.exmath.2022.03.002>
- 2022 **Greatest common divisors of shifted primes and Fibonacci numbers**
(with Abhishek Jha)
Research in Number Theory **8**, 65
<https://doi.org/10.1007/s40993-022-00365-2>

- 2022 **Practical complexities of probabilistic algorithms for solving Boolean polynomial systems**
(with Stefano Barbero, Emanuele Bellini, and Javier Verbel)
Discrete Applied Mathematics **309**, 13–31
<https://doi.org/10.1016/j.dam.2021.11.014>
- 2022 **On the divisibility of the rank of appearance of a Lucas sequence**
International Journal of Number Theory **18**, 2145–2156
<https://doi.org/10.1142/S1793042122501093>
- 2022 **On the number of residues of linear recurrences**
Research in Number Theory **8**, 7
<https://doi.org/10.1007/s40993-021-00305-6>
- 2022 **The largest entry in the inverse of a Vandermonde matrix**
(with Jeffrey Shallit and Shun Zhang)
Linear and Multilinear Algebra **70**, 5634–5641
<https://doi.org/10.1080/03081087.2021.1922337>
- 2022 **Membership in random ratio sets**
Indagationes Mathematicae **33**, 1326–1333
<https://doi.org/10.1016/j.indag.2022.08.007>
- 2022 **On the l.c.m. of shifted Lucas numbers**
Indagationes Mathematicae **33**, 1001–1011
<https://doi.org/10.1016/j.indag.2022.04.006>
- 2022 **On the exponents in the factorizations of r consecutive numbers**
Quaestiones Mathematicae **45**, 1221–1228
<https://doi.org/10.2989/16073606.2021.1938277>
- 2022 **Practical central binomial coefficients**
Quaestiones Mathematicae **44**, 1141–1144
<https://doi.org/10.2989/16073606.2020.1775156>
- 2021 **A note on the natural density of product sets**
(with Sandro Bettin and Dimitris Koukoulopoulos)
Bulletin of the London Mathematical Society **53**, 1407–1413
<https://doi.org/10.1112/blms.12506>
- 2021 **Additive bases and Niven numbers**
Bulletin of the Australian Mathematical Society **104**, 373–380
<https://doi.org/10.1017/S0004972721000186>
- 2021 **On the l.c.m. of shifted Fibonacci numbers**
International Journal of Number Theory **17**, 2009–2018
<https://doi.org/10.1142/S1793042121500743>
- 2021 **On the least common multiple of shifted powers**
Journal of Integer Sequences **24**, article 21.7.3
<https://cs.uwaterloo.ca/journals/JIS/VOL24/Sanna/sanna7.html>
- 2021 **On the least common multiple of random q -integers**
Research in Number Theory **7**, 16
<https://doi.org/10.1007/s40993-021-00242-4>

- 2021 **On the condition number of the Vandermonde matrix of the n th cyclotomic polynomial**
(with Antonio J. Di Scala and Edoardo Signorini)
Journal of Mathematical Cryptology **15**, 174–178
<https://doi.org/10.1515/jmc-2020-0009>
- 2020 **Greedy approximations by signed harmonic sums and the Thue–Morse sequence**
(with Sandro Bettin and Giuseppe Molteni)
Advances in Mathematics **366**, 107068
<https://doi.org/10.1016/j.aim.2020.107068>
- 2020 **On the l.c.m. of random terms of binary recurrence sequences**
Journal of Number Theory **213**, 221–231
<https://doi.org/10.1016/j.jnt.2019.12.004>
- 2020 **A note on product sets of random sets**
Acta Mathematica Hungarica **162**, 76–83
<https://doi.org/10.1007/s10474-019-01014-4>
- 2020 **On numbers divisible by the product of their nonzero base b digits**
Quaestiones Mathematicae **43**, 1563–1571
<https://doi.org/10.2989/16073606.2019.1637956>
- 2020 **Least common multiple of polynomial sequences**
(with Danilo Bazzanella)
Rendiconti del Seminario Matematico Università e Politecnico di Torino **78**, n° 1, 21–25
- 2020 **Directions sets: A generalization of ratio sets**
(with Paolo Leonetti)
Bulletin of the Australian Mathematical Society **101**, 389–395
<https://doi.org/10.1017/S0004972719000959>
- 2020 **Practical numbers among the binomial coefficients**
(with Paolo Leonetti)
Journal of Number Theory **207**, 145–155
<https://doi.org/10.1016/j.jnt.2019.07.005>
- 2020 **On the number of distinct exponents in the prime factorization of an integer**
Proceedings of the Indian Academy of Sciences **130**, 27
<https://doi.org/10.1007/s12044-020-0556-y>
- 2020 **p -adic denseness of members of partitions of \mathbb{N} and their ratio sets**
(with Piotr Miska)
Bulletin of the Malaysian Mathematical Sciences Society **43**, 1127–1133
<https://doi.org/10.1007/s40840-019-00728-6>
- 2019 **Waring’s theorem for binary powers**
(with Daniel Kane and Jeffrey Shallit)
Combinatorica **39**, 1335–1350
<https://doi.org/10.1007/s00493-019-3933-3>
- 2019 **Practical numbers in Lucas sequences**
Quaestiones Mathematicae **42**, 977–983
<https://doi.org/10.2989/16073606.2018.1502697>

- 2019 **On the p -adic denseness of the quotient set of a polynomial image**
(with Piotr Miska and Nadir Murru)
Journal of Number Theory **197**, 218–227
<https://doi.org/10.1016/j.jnt.2018.08.010>
- 2019 **On numbers n with polynomial image coprime with the n th term of a linear recurrence**
(with Daniele Mastrostefano)
Bulletin of the Australian Mathematical Society **99**, 23–33
<https://doi.org/10.1017/S0004972718000606>
- 2019 **On numbers n relatively prime to the n th term of a linear recurrence**
Bulletin of the Malaysian Mathematical Sciences Society **42**, 827–833
<https://doi.org/10.1007/s40840-017-0514-8>
- 2018 **Small values of signed harmonic sums**
(with Sandro Bettin and Giuseppe Molteni)
Comptes Rendus Mathématique **356**, 1062–1074
<https://doi.org/10.1016/j.crma.2018.11.007>
- 2018 **On the greatest common divisor of n and the n th Fibonacci number**
(with Paolo Leonetti)
Rocky Mountain Journal of Mathematics **48**, 1191–1199
<https://doi.org/10.1216/RMJ-2018-48-4-1191>
- 2018 **The density of numbers n having a prescribed G.C.D. with the n th Fibonacci number**
(with Emanuele Tron)
Indagationes Mathematicae **29**, 972–980
<https://doi.org/10.1016/j.indag.2018.03.002>
- 2018 **The moments of the logarithm of a G.C.D. related to Lucas sequences**
Journal of Number Theory **191**, 305–315
<https://doi.org/10.1016/j.jnt.2018.03.012>
- 2018 **A note on primes in certain residue classes**
(with Paolo Leonetti)
International Journal of Number Theory **14**, 2219–2223
<https://doi.org/10.1142/S1793042118501336>
- 2018 **Central binomial coefficients divisible by or coprime to their indices**
International Journal of Number Theory **14**, 1135–1141
<https://doi.org/10.1142/S1793042118500707>
- 2018 **On the k -regularity of the k -adic valuation of Lucas sequences**
(with Nadir Murru)
Journal de Théorie des Nombres de Bordeaux **30**, 227–237
<https://doi.org/10.5802/jtnb.1025>
- 2017 **On the closure of the image of the generalized divisor function**
Uniform Distribution Theory **12**, 77–90
<https://doi.org/10.1515/udt-2017-0016>
- 2017 **A coprimality condition on consecutive values of polynomials**
(with Márton Szikszai)
Bulletin of the London Mathematical Society **49**, 908–915
<https://doi.org/10.1112/blms.12078>

- 2017 **Distribution of integral values for the ratio of two linear recurrences**
Journal of Number Theory **180**, 195–207
<https://doi.org/10.1016/j.jnt.2017.04.015>
- 2017 **p -adic quotient sets**
 (with Stephan Ramon Garcia, Yu Xuan Hong, Florian Luca, Elena Pinsker,
 Evan Schechter, and Adam Starr)
Acta Arithmetica **179**, 163–184
<https://doi.org/10.4064/aa8579-4-2017>
- 2017 **The quotient set of k -generalised Fibonacci numbers is dense in \mathbb{Q}_p**
Bulletin of the Australian Mathematical Society **96**, 24–29
<https://doi.org/10.1017/S0004972716001118>
- 2017 **A factor of integer polynomials with minimal integrals**
Journal de Théorie des Nombres de Bordeaux **29**, 637–646
<https://doi.org/10.5802/jtnb.994>
- 2017 **On numbers n dividing the n th term of a Lucas sequence**
International Journal of Number Theory **13**, 725–734
<https://doi.org/10.1142/S1793042117500373>
- 2017 **On the p -adic valuation of Stirling numbers of the first kind**
 (with Paolo Leonetti)
Acta Mathematica Hungarica **151**, 217–231
<https://doi.org/10.1007/s10474-016-0680-4>
- 2016 **The p -adic valuation of Lucas sequences**
The Fibonacci Quarterly **54**, 118–124
<https://www.fq.math.ca/54-2>
- 2016 **On the p -adic valuation of harmonic numbers**
Journal of Number Theory **166**, 41–46
<https://doi.org/10.1016/j.jnt.2016.02.020>
- 2015 **Counting arithmetic formulas**
 (with Edinah K. Gnang and Maksym Radziwiłł)
European Journal of Combinatorics **47**, 40–53
<https://doi.org/10.1016/j.ejc.2015.01.007>
- 2015 **On the number of arithmetic formulas**
International Journal of Number Theory **11**, 1099–1106
<https://doi.org/10.1142/S1793042115500591>
- 2015 **On the sum of digits of the factorial**
Journal of Number Theory **147**, 836–841
<https://doi.org/10.1016/j.jnt.2014.09.003>
- 2014 **Covering an arithmetic progression with geometric progressions and vice versa**
International Journal of Number Theory **10**, 1577–1582
<https://doi.org/10.1142/S1793042114500456>
- 2014 **On the exponential sum with the sum of digits of hereditary base b notation**
INTEGERS **14**, article A36
<https://www.integers-ejcnt.org/vol14.html>

- 2014 **On the asymptotic density of the support of a Dirichlet convolution**
Journal of Number Theory **134**, 1–12
<https://doi.org/10.1016/j.jnt.2013.07.012>
- 2013 **Uncertainty principles connected with the Möbius inversion formula**
 (with Paul Pollack)
Bulletin of the Australian Mathematical Society **88**, 460–472
<https://doi.org/10.1017/S0004972712001128>
- 2012 **On arithmetic progressions of integers with a distinct sum of digits**
Journal of Integer Sequences **15**, article 12.8.1
<https://cs.uwaterloo.ca/journals/JIS/VOL15/Sanna/sanna3.html>
- 2012 **A new elementary proof of the inequality $\varphi(n) > \pi(n)$**
Notes on Number Theory and Discrete Mathematics **18**, No. 3, 35–37
<https://nntdm.net/volume-18-2012/number-3/35-37>

References

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